Nonlinear Shell Finite Elements

Robert Winkler

1st Workshop
on Nonlinear Analysis of Shell Structures

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University of Innsbruck, Faculty of Civil Engineering
University of Innsbruck, Faculty of Mathematics, Informatics and Physics

Natters/Tyrol, 15/06/2010
Introduction

f77 subdivision

- Franz-Josef Falkner Stability
- Roland Traxl Material
- Stefan Schett Solid shell elements
f77 subdivision

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Outline

- The role of the element formulation
- Current developments and ...
- their impact on (near future) applications
Ill-posed continuous problem due to extreme slenderness
Challenge

Ill-posed continuous problem due to extreme slenderness
→ numerical difficulties treating the discretized structure
Challenge

Ill-posed continuous problem due to extreme slenderness
→ numerical difficulties treating the discretized structure

- Membrane behavior
  - Shear locking
  - Parallelogram locking
  - Trapezoidal locking
  - Poisson failure
  - Normal strain failure

- Bending behavior
  - Transverse shear locking
  - Membrane locking

- Additionally (solid shell elements only)
  - Volumetric locking
  - Pinching locking (‘Curvature thickness locking’)
  - Transverse Poisson failure (‘Poisson thickness locking’)
  - Transverse normal strain failure
Abaqus S4R/S4 large strain shell elements (e.g.)

- late 1990’s
- Resultant based (Simo & Fox 1989)
- Penalized drill rotation (Fox & Simo 1992)
- S4R: Reduced integration (Belytschko et al. 1992)
- S4: Assumed \textit{enhanced} membrane strains (Betsch et al. 1996)
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General limits of perfectibility
- Trapezoidal locking in bilinear (4 node) elements
- Membrane locking in quadratic (8/9 node) elements
Problems

- S4R: 3 algorithmic parameter
  - HOURGLASS STIFFNESS
- S4: sightly less robust, still 1 algo. parameter

For thin to moderately thick, *smooth*, and *homogeneous* shells there are no substantial improvements to be expected...
**Problems**

- S4R: 3 algorithmic parameter
  - *HOURGLASS STIFFNESS*, ..., *
- S4: sightly less robust, still 1 algo. parameter

For thin to moderately thick, *smooth*, and *homogeneous* shells there are no substantial improvements to be expected...

**Even though, what can we do better?**

- Getting rid of any algo. parameter (done...)
- Improved treatment of shell intersections (stiffened structures)
- Improved transverse shear distribution (layered structures)
Shell theory attempts the impossible: to provide a two-dimensional representation of an intrinsically three-dimensional phenomenon (Koiter & Simmonds 1972)
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4 node quadrilateral shell element
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4 node quadrilateral shell element

8 node hexahedral *solid* shell element
Solid Shell Elements

- **Basics**
  - Nodes on bottom and top surfaces, 3 D.O.F.s, each
  - 3D stress state, explicit thickness change

- **Pros**
  - Arbitrary 3D constitutive laws
  - Unlimited rotations
  - Large strains

- **Cons**
  - Mesh depends on thickness
  - Increased number of overall D.O.F.s by factor (e.g.)
    \[ (1 + \frac{1}{n})(1 + \frac{1}{m}) \sim 1.56 \]
  - Extra ill-conditioning (dynamics, iterative solvers)
    \[ \text{cond } K^e \sim \left( \frac{h}{t} \right)^4 \]
Shell Intersections

- **How to treat the joint section?**
  - Conventional solid elements
  - Solid shell elements (which is the thickness direction?)
  - Solid beam elements
Shell Intersections

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  - Conventional solid elements
  - Solid shell elements (which is the thickness direction?)
  - Solid beam elements

- Increased efforts for mesh generation...
- Use AAR elements!
  - Arbitrary aspect ratios
  - Inadequate for layered structures
  - Increased stress oscillations
4 node quadrilateral shell elements QS4A8E5..11

- Prototype: Bischoff & Ramm (1997)
- Matlab, Abaqus (UEL), DIANA
- Research tool: RI, SRI, WRI, ACMS
- Bifurcation analysis (analytical tangent)
- Limited applicability (stiffened structures)
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8 node hexahedral shell elements HS8A8E8..
- Prototype: Klinkel et al. (1999)
- Passes in-plane and bending patch test
- 'Optimal' solid shell element for layered structures
- Matlab, Abaqus (UEL) $\rightarrow$ 'small launcher' ($RT$)
ACOSTA Work Package (selected items)

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- **8 node hexahedral continuum elements HC8E18..21**
  - Prototype: Alves de Souza et al. (2003)
  - Significantly improved version
  - Passes in-plane and volumetric patch test, pinching locking!
  - Matlab, Abaqus (UEL) → aniso. large strain plasticity (RT)
8 node hexahedral continuum element HC8A18E9

- Arbitrary Aspect Ratios (AAR)
- 'Optimal' formulation wrt. locking and shape sensitivity
- Fails patch tests (increased stress oscillations)
- Matlab, Abaqus (UEL) → 'small launcher' (RT)
8 node hexahedral continuum element HC8A18E9
- Arbitrary Aspect Ratios (AAR)
- 'Optimal' formulation wrt. locking and shape sensitivity
- Fails patch tests (increased stress oscillations)
- Matlab, Abaqus (UEL) → 'small launcher' (RT)

8 node hexahedral beam elements HB8A4E7..11
- 'Solid beam' elements
- Modeling joint sections, solid-beam transitions, etc.
- Matlab, Abaqus (UEL)
Selected Results

Standard benchmark ‘Pinched hemisphere’

<table>
<thead>
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<th>material</th>
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<tbody>
<tr>
<td>( E = 6.825 \cdot 10^7 )</td>
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<tr>
<td>( \nu = 0.3 )</td>
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<td>( R = 10 )</td>
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</tr>
<tr>
<td>( t = 0.04 )</td>
<td></td>
</tr>
<tr>
<td>( F = 200 )</td>
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</table>
Pinched hemisphere, $d=0.04$
Pinched hemisphere, d=0.01
C-section cantilever beam

<table>
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<tr>
<th>material</th>
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<tbody>
<tr>
<td>( E = 1 \cdot 10^7 )</td>
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<tr>
<td>( \nu = 0.3333 )</td>
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<table>
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<tr>
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<tbody>
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<td>( L = 36 )</td>
<td></td>
</tr>
<tr>
<td>( t = 0.05 )</td>
<td></td>
</tr>
<tr>
<td>( b = 2.025 )</td>
<td></td>
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<tr>
<td>( h = 6.05 )</td>
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Um die Effizienz der Elemente im Bezug auf das Beispiel besser zu untersuchen ist eine Netzverfeinerung vorgenommen worden. Netz 1 beinhaltet nach Querschnittsdiscretisierung nach Abb. (??) 10(a) oder 8(b) Elemente über den Querschnitt und 36 Elemente über die Trägerlänge, somit ergeben sich eine Vernetzung beim Netz 1 von 360(10x36) Elemente für (a) und 288(8x36) Elemente für (b). Netz 2 wurden in Bezug auf Netz 1 um die Hälfte verfeinert und so ergeben sich für Netz 2 1440(20x72) Elemente für (a) und 1296(18x72) Elemente für (b). Als Ergebnis wird die Verschiebung des Belastungspunktes in die Belastungsrichtung nach Belastungsgöße in Abb. (??) für Netz 1 und Abb. (??) für Netz 2 dargestellt.
C-section cantilever beam, coarse mesh
Selected Results

C-section cantilever beam, fine mesh

absolute displacement of the loaded node

external load level

QS4A8E9
S4R
SC8RC
SC8R
HC8E21C
HC8E21
HC8E18C
HC8E18
HS8A8E9C
HS8A8E9
HC8A18E9C
HC8A18E9
S4R
SC8RC

Robert Winkler (University of Innsbruck)
I-section cantilever beam

<table>
<thead>
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<th>Material</th>
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<tbody>
<tr>
<td>$E = 2 \cdot 10^6$</td>
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<td>$\nu = 0.30$</td>
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<table>
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<td>$L = 48$</td>
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<tr>
<td>$t = 0.25$</td>
</tr>
<tr>
<td>$b = 3$</td>
</tr>
<tr>
<td>$h = 3$</td>
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</table>

<table>
<thead>
<tr>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F = \lambda \cdot 1$</td>
</tr>
<tr>
<td>$F_h = F/1000$</td>
</tr>
</tbody>
</table>
Results

I-section cantilever beam

- SC8R – A
- SC8R – B
- SC8R – C
- HC8E18 – A
- HC8E18 – B
- HC8E18 – C
- HC8E21 – A
- HC8E21 – B
- HC8E21 – C
- C3D8 – C
- C3D8 – D
- HS8A8E9 – A
- HS8A8E9 – B
- HS8A8E9 – C
- HC8A18E9 – A
- HC8A18E9 – B
- HC8A18E9 – C

Absolute vertical displacement of the loaded node

External load level
Lessons learned

- Solid shell elements are a valuable tool for...
  - strength analyses (shell-solid transitions etc.)
  - large strain and/or contact problems (metal forming etc.)
  - delamination...
  - Conditioning problem in implicit dynamics
- Conventional shell elements should be preferred for
  - stability analyses
  - large-scale applications
  - ... whenever feasible!
- Rigid joints in multi-shell structures
  - Feasible approximation in most stability analyses
  - Deteriorating in the large displacement/strain regime
- Mesh quality cannot be judged utilizing linear results!
• Winkler “Three dimensional shell elements for industrial applications” CEAS Berlin 2007

• Winkler & Falkner “Numerical stability analyses of shell structures, revisited” SSTA Jurata 2009

• Winkler “Two comments on membrane locking” GAMM Karlsruhe 2010

• Winkler, Traxl, Schett “Large strain continuum elements with arbitrary aspect ratios” ECCM Paris 2010

• Winkler, Traxl, Schett “Low-order continuum shell elements at finite strains, their limits of perfectibility and applications” SolMech Warshaw 2010