Anisotropic Elastoplasticity at Finite Strains: Formulation and Numerical Implementation

1^{st} Workshop
on Nonlinear Analysis of Shell Structures

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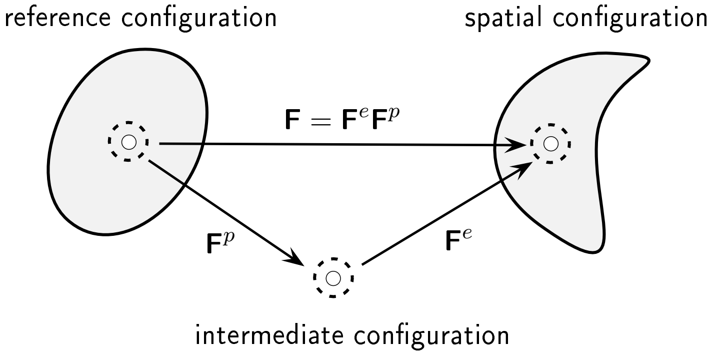
Object

Implementation of a constitutive model

- Examination of influence of anisotropy
- Performing cross checks
- Unlimited access to state variables

Requirements for the material model:

- Elastoplastic behaviour
- Anisotropy
- Moderate strains
- Upgradeability



Multiplicative decomposition of deformation gradient not unique

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^p = \mathbf{F}^e\mathbf{Q}\mathbf{Q}^T\mathbf{F}^p = \tilde{\mathbf{F}}^e\tilde{\mathbf{F}}^p$$

Basic Framework

Concepts with assumptions of the rotational part of $\mathbf{F}^p$

- Eidel and Gruttmann, 2003 (wrt intermediate configuration)
- Sansour, 2008 (wrt reference configuration)

Concepts in which the rotational part of $\mathbf{F}^p$ is neglected

- Miehe, 2002 (additive approach, wrt reference configuration)

Assumption: existence of a plastic metric $\mathbf{G}^p$

$$\mathbf{C} = \mathbf{F}^T\mathbf{F}$$
$$\mathbf{G}^p = \mathbf{F}^{p\ T}\mathbf{F}^p$$

Additive decomposition of strain measure

$$\mathbf{E}(\mathbf{C}) = \mathbf{E}^e + \mathbf{E}^p(\mathbf{G}^p)$$

Strain measure: Logarithmic strains (Lagrangian)

$$\mathbf{E}(\mathbf{C}) = \frac{1}{2}\ln\ \mathbf{C}$$
$$\mathbf{E}^p(\mathbf{G}^p) = \frac{1}{2}\ln\ \mathbf{G}^p$$

Features and Advantages:

- At isotropy and with logarithmic strains:

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^p \quad \Leftrightarrow \quad \mathbf{E}(\mathbf{C}) = \mathbf{E}^e + \mathbf{E}^p(\mathbf{G}^p)$$

- Concepts from small strain theory can be used
- Volume preserving behaviour assured

Additive Approach

Modular structure for implementation:

Geometric Preprocessor ⇒ Model ⇒ Geometric Postprocessor

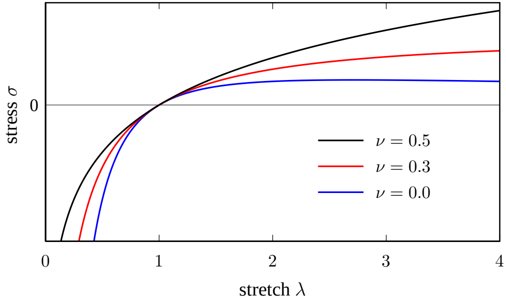
↑

Applicability of Algorithms from the small strain theory

- Isotropic:
    - Hencky Material
    - von Mises flow criterion
- Anisotropic:
    - anisotropic Hencky-type Material
    - Hoffman-Hill criterion

Hencky Material:
Hyperelastic constitutive model based on logarithmic strains

$$\boldsymbol{\tau} = \frac{\partial \psi}{\partial \boldsymbol{\epsilon}} = \mathbb{C} : \boldsymbol{\epsilon}$$



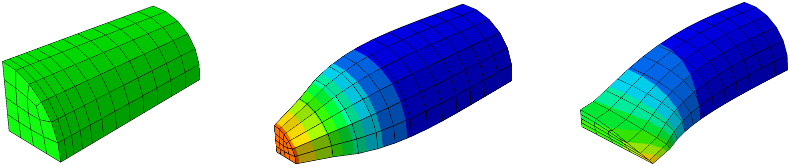
Based on the **Hoffman-Hill** criterion

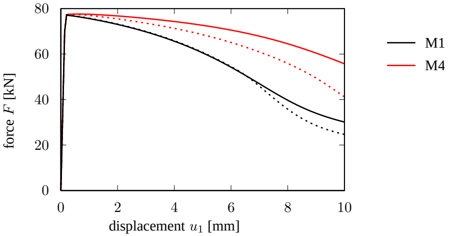
$$\Phi(\boldsymbol{\tau}, \overline{\sigma}_y) = \frac{1}{2}\boldsymbol{\tau}^T \mathbf{P} \boldsymbol{\tau} + \mathbf{q}^T \boldsymbol{\tau} - \overline{\sigma}_y^2$$

- Anisotropic extension of the von Mises criterion
- Distinction between tension and compression

- in a FE program in MATLAB
- as a User Material in Abaqus (Fortran)

Example: Necking of a rod





Comparison with Abaqus Material

- Similar results for plasticity without hardening
- Differences at hardening and anisotropy
- Differences for big elastic deformations

Summary

- Anisotropic constitutive model with tension compression distinction
- Additive decomposition of logarithmic strains
- Algorithm from small strain theory can be used
  - Return mapping
  - Analytical determination of Jacobian
  - Upgradeability
- Adequate approach for moderate strains