Anisotropic Elastoplasticity at Finite Strains: Formulation and Numerical Implementation

1st Workshop on Nonlinear Analysis of Shell Structures

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Implementation of a constitutive model

- Examination of influence of anisotropy
- Performing cross checks
- Unlimited access to state variables

Requirements for the material model:

- Elastoplastic behaviour
- Anisotropy
- Moderate strains
- Upgradeability
Multiplicative decomposition of deformation gradient not unique

\[ F = F^e F^p = F^e QQ^T F^p = \tilde{F}^e \tilde{F}^p \]
Basic Framework

Concepts with assumptions of the rotational part of $F^p$

- Eidel and Gruttmann, 2003 (wrt intermediate configuration)
- Sansour, 2008 (wrt reference configuration)

Concepts in which the rotational part of $F^p$ is neglected

- Miehe, 2002 (additive approach, wrt reference configuration)
Additive Approach

Assumption: existence of a plastic metric $G^p$

$$C = F^T F$$

$$G^p = F^p \, T \, F^p$$

Additive decomposition of strain measure

$$E(C) = E^e + E^p(G^p)$$

Strain measure: Logarithmic strains (Lagrangian)

$$E(C) = \frac{1}{2} \ln \, C$$

$$E^p(G^p) = \frac{1}{2} \ln \, G^p$$
Features and Advantages:

- At isotropy and with logarithmic strains:
  \[ F = F^e F^p \iff E(C) = E^e + E^p(G^p) \]
- Concepts from small strain theory can be used
- Volume preserving behaviour assured
Modular structure for implementation:

Geometric Preprocessor $\Rightarrow$ Model $\Rightarrow$ Geometric Postprocessor

Applicability of Algorithms from the small strain theory
Constitutive Models

- Isotropic:
  - Hencky Material
  - von Mises flow criterion

- Anisotropic:
  - anisotropic Hencky-type Material
  - Hoffman-Hill criterion
Hencky Material:
Hyperelastic constitutive model based on logarithmic strains

\[ \mathbf{T} = \frac{\partial \psi}{\partial \mathbf{\epsilon}} = \mathbf{C} : \mathbf{\epsilon} \]
Based on the Hoffman-Hill criterion

\[ \Phi(\tau, \sigma_y) = \frac{1}{2} \tau^T P \tau + q^T \tau - \sigma_y^2 \]

- Anisotropic extension of the von Mises criterion
- Distinction between tension and compression
Implementation

- in a FE program in MATLAB
- as a User Material in Abaqus (Fortran)
Example: Necking of a rod

Roland Traxl (University of Innsbruck)
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IMPLEMENTATION

Comparison with Abaqus Material

- Similar results for plasticity without hardening
- Differences at hardening and anisotropy
- Differences for big elastic deformations
Summary

- Anisotropic constitutive model with tension compression distinction
- Additive decomposition of logarithmic strains
- Algorithm from small strain theory can be used
  - Return mapping
  - Analytical determination of Jacobian
  - Upgradeability
- Adequate approach for moderate strains