Monte Carlo Methods in Iterative Solvers

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1st Workshop on Nonlinear Analysis of Shell Structures

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Reliability and Sensitivity

Tasks

- Development of new tools for reliability analysis
- Reducing coputation cost of sensitivity analysis

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Main Activities

- Development of numerical methods for Monte Carlo based sensitivity analyis in combination with FE-analysis
- Implementation of Monte Carlo simulation methods in combination with iterative methods
- Investigation of sensitivity with respect to input variables

Monte Carlo Based Sensitivity Analysis

Approach

- Generation of input sample with certain distributions (i.e. uniform distributions)
- Calculation of the model response for each of these input values
- Inspection of the distribution of output data (and its correlation to the input data)

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Challenges

- Relies on multiple model calls
- Expensive computational cost if the model itself requires costly computations

Linear FE Models

Idea

- Small changes in input parameters result in neighbouring equations
- Use data extracted from previous calculation for a speed up

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Approaches to use data from previous solutions

- iterative scheme
- preconditioned conjugated gradients (pcg)

PCG

Ideas

- Use the Cholesky decomposition of the system matrix $K = LL^T$ for preconditioning subsequent equations
- Use a previously calculated solution as the starting vector

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Performance (computation time)

preconditioner	DOF	$spr = 1\%$	$spr = 5\%$	$spr = 10\%$
IC with tol=1e-2	5642	0.6570	0.6727	0.6919
cond with cholesky neighbour	5642	0.1167	0.1551	0.1615

Nonlinear FE Models

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- Start the FE computation using the nominal parameter values
- At some step n_{var}, change the input parameters and perform additional equilibrium iterations
- Continue with applying load steps until the target load is reached

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Difficulties

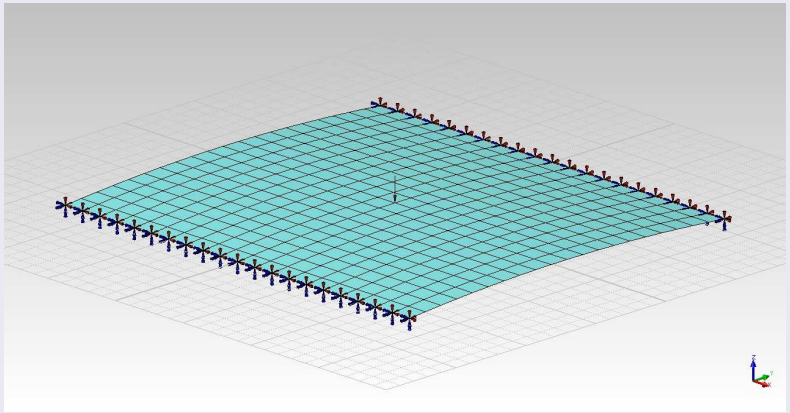
- Numerical solution of the problem may be path dependent
- Speed of convergence can be affected

Model: Shallow Cylindrical Roof

Model

Model input: F,E,ν

Model output: displacement in middle node



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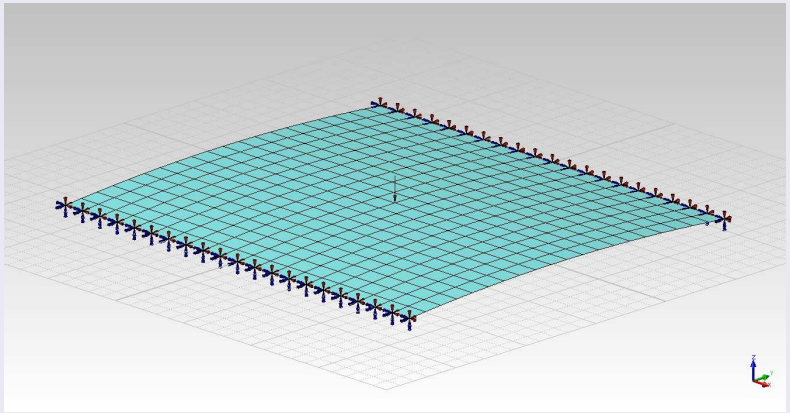
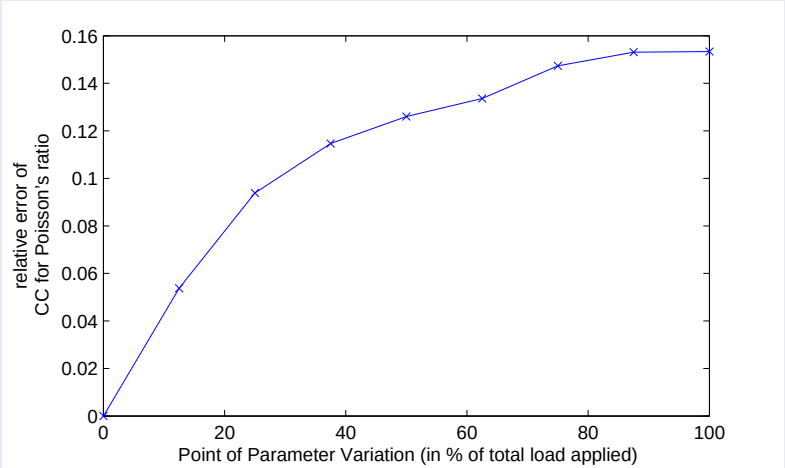


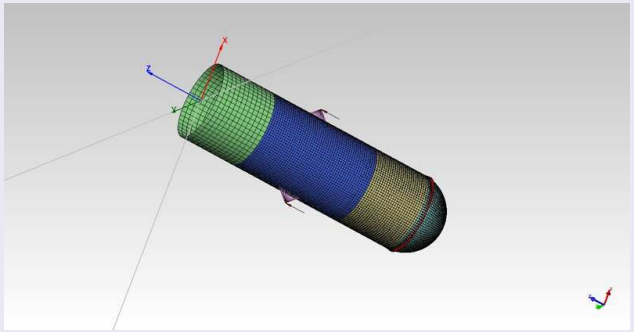
Result

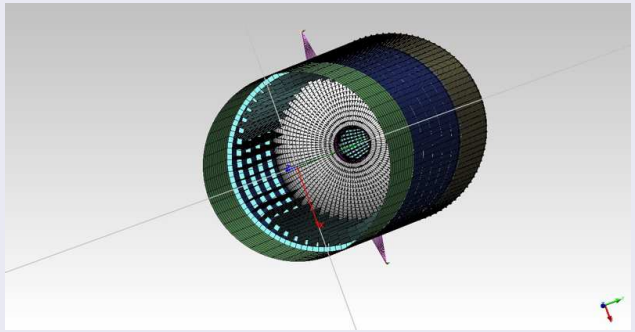
- Correlation indicators change
- Ranking of indicators stays the same
- Significantly faster computation



Simplified Launcher Model

Input variables





	Description of input parameters	
i	Parameter X_i	Mean μ_i
1	hydrostatic pressure cylinder3	0.4 MPa
2	hydrostatic pressure sphere1	0.4 MPa
3	hydrostatic pressure sphere2	0.4 MPa
4	aerodynamic pressure	-0.05 MPa
5	booster loads y-direction node1	40000 N
6	booster loads y-direction node2	20000 N
7	booster loads z-direction node1	$3.e6$ N
8	booster loads z-direction node2	$1.e6$ N
9	mechanical loads x-direction	100 N
10	mechanical loads y-direction	50 N
11	mechanical loads z-direction	300 N
Output: Load Proportionality Factor (LPF)		

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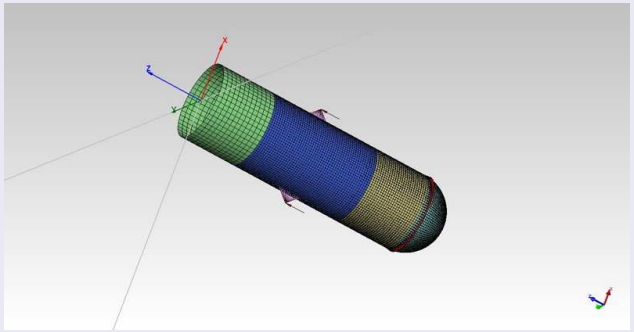

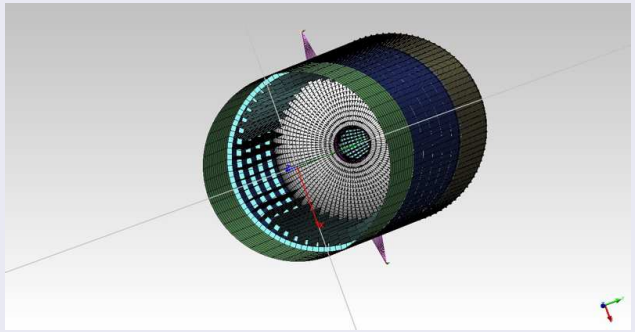

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Input:

Uniform distributions with spread $\pm 15\%$ of nominal value

Simplified Launcher Model

Results

- Correlation indicators change in dependance of the point of parameter variation
- Noticeable speed up

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Computation cost

Point of parameter variation [as fraction of target load]	processing time [in 10^6 s]
0	5.801
0.5	3.947
1.0	3.587

Simplified Launcher Model

Sensitivities

	Variation at beginning (load factor = 0.0)				Variation at load factor = 1.0			
i	CC	SRC	RCC	SRCC	CC	SRC	RCC	SRCC
1	0.1113	0.1608	0.0556	0.2873	-0.0363	-0.0049	-0.0226	0.2252
2	-0.0102	0.1114	-0.0075	0.2550	-0.1187	-0.0158	-0.0902	0.1762
3	0.1207	0.1658	0.1083	0.2469	-0.0547	0.0083	-0.0406	0.1249
4	0.0011	0.0418	0.0526	0.2442	-0.0368	-0.0037	-0.0030	0.2086
5	-0.0548	-0.0021	-0.0737	0.1249	-0.0709	-0.0126	-0.0632	0.1733
6	-0.0664	-0.0566	-0.0932	0.0381	-0.0373	0.0061	-0.0436	0.1517
7	-0.9582	-0.9841	-0.9624	-0.8221	-0.9972	-0.9958	-0.9999	-0.8268
8	-0.0344	-0.0400	-0.0436	0.0683	-0.0055	0.0129	0.0030	0.1385
9	0.0112	0.0084	0.0647	0.2252	0.0124	-0.0132	0.0165	0.1764
10	0.0032	-0.0017	0.0150	0.1737	-0.0221	-0.0281	0.0015	0.1840
11	-0.0176	-0.0550	-0.0406	0.1025	-0.0002	0.0020	0.0045	0.2037

Outlook

Limitations

- Evaluation of smaller sensitivities is difficult due to errors
- Model evaluation for frontskirt model still time intensive

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Future Possibilities

- Further development of the presented algorithms
- Embedding algorithms into FE framework