

# Quantitative assessment of random field models in FE buckling analyses of composite cylinders

## 2<sup>nd</sup> Workshop on Structural Analysis of Lightweight Structures

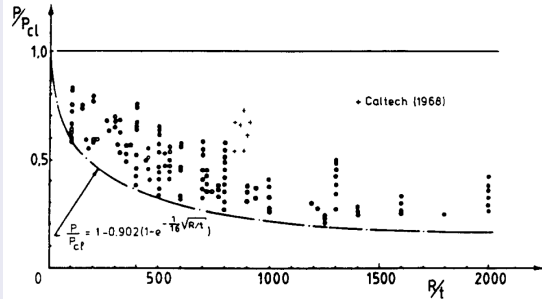
University of Innsbruck, Faculty of Civil Engineering Sciences

Natters/Tyrol, 30/05/2012

- Motivation
- Random Fields
- Geometrical Imperfection Measurements
- Results
- Conclusions

# MOTIVATION - WHY?

## LARGE DISCREPANCIES



## OBSERVATIONS

- Effect of imperfections.
- Variation of the buckling load.

## KARHUNEN-LOÈVE EXPANSION

$$U(\mathbf{x}, \omega) = g(\mathbf{x}) + \sum_{n=0}^N \sqrt{\lambda_n} \xi_n(\omega) \phi_n(\mathbf{x}) \quad (1)$$

Basically, a (truncated) summation of eigenvectors with random weights.

## COVARIANCE MATRIX

- Contains the imperfection structure of the field.
- Eigenvalue decomposition gives eigenvalues and eigenvectors for the KL-expansion.

## AUTO-COVARIANCE FUNCTION

- Fit to measurement data.
- Reproduce a positive definite matrix.

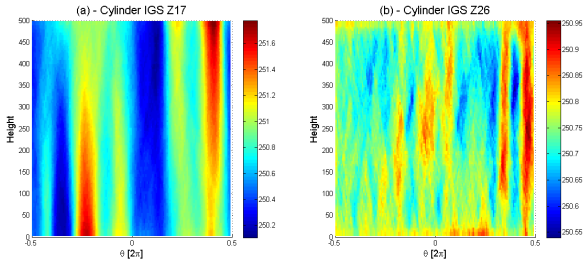
## PRINCIPAL COMPONENT ANALYSIS (PCA)

- Assemble covariance matrix directly from measurement data.
- $C(x_i, x_j) = E(U(x_i), U(x_j)) - \mu_i \mu_j$

## COMPOSITE CYLINDERS

- 10 available samples.
- Very thin:  $\frac{R}{t} = 1000$ .
- Very fine imperfection measurements.
- Data provided by DLR<sup>1</sup>.

[1] R. Degenhardt et al., Investigations on imperfection sensitivity and deduction of improved knock-down factors for unstiffened CFRP cylindrical shells. *Composite Structures*, 92(8):1939–1946, 2010



## INVESTIGATION OF THE IMPERFECTIONS

- Mean is constant.
- Gaussian distribution.
- Standard deviation: 2 sets.
- Correlation structure: 2 sets.
- 1 outlier identified: deleted from the sample.

## WE CONSIDER:

- Composite cylinders.
- Very small sample size.
- Large variation between the different samples.
- Limited knowledge of the production process.



# IMPORTANT REMARKS



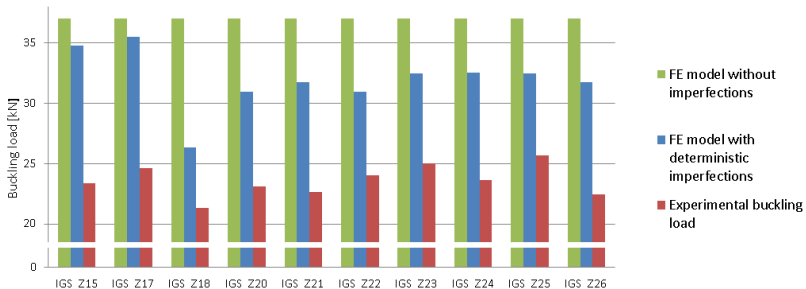
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## RANDOM FIELD EVALUATION

- Different causes for discrepancies (model, boundaries, loads, etc.).
- Compare to FE failure load of the measured imperfections.

# EVALUATION OF AUTOCOVARANCE FUNCTION



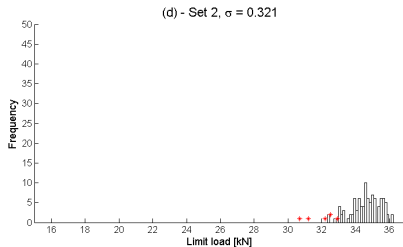
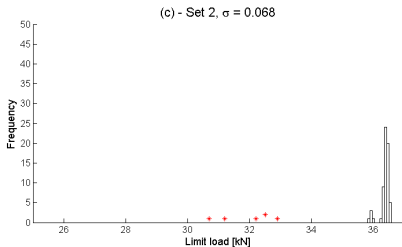
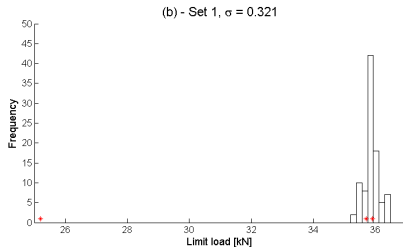
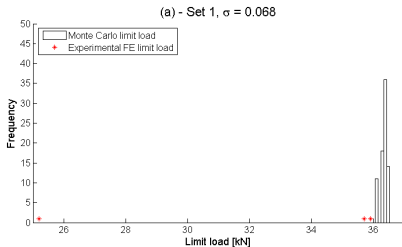
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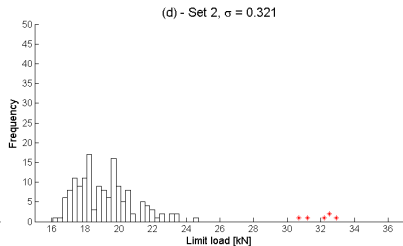
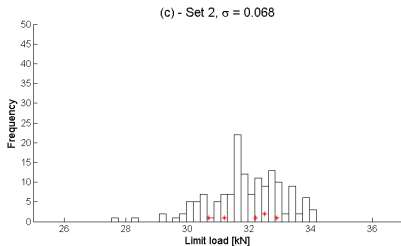
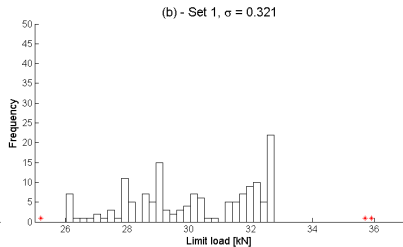
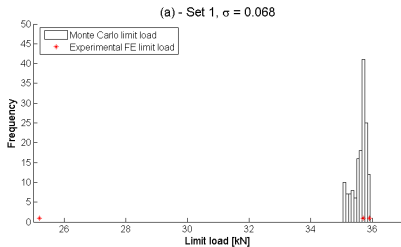
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# EVALUATION OF PCA



# SENSITIVITY OF THE STANDARD DEVIATION



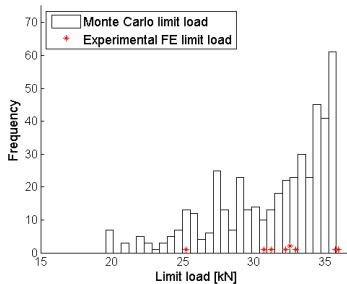
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- Standard deviations: 0.321, 0.152, 0.068.
- All Principal Components included.

## MODEL EVALUATION

- Evaluate the validity of models.
- Isolate the random field model effects from other imperfections.

## MODELS

- Autocovariance function: overestimation of the failure load.
- PCA: samplesize should be large enough.
- Do not underestimate the effect of standard deviation.