Stochastic parameter calibration in a finite element model for a lightweight structure

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5th Workshop on structural analysis of lightweight structures

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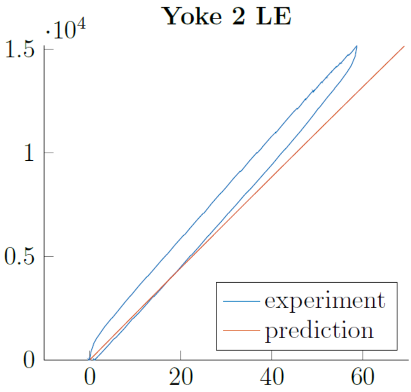
Motivation

- **FE-Model**
  - Developed by INTALES
  - Degrees of freedom: $\sim 10^6$
  - Calculation time: $\sim$ 11 hours
- **Actuator modifications**
  - Connector elements
  - Kelvin-Voigt model



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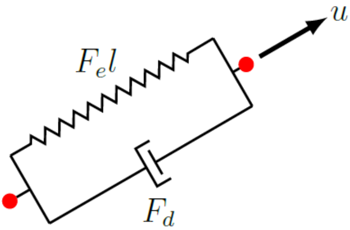
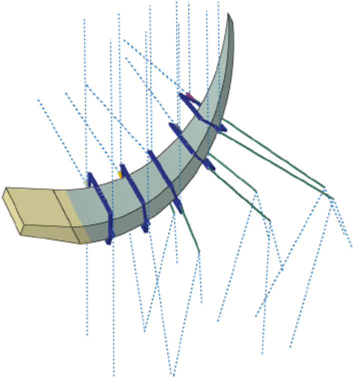


Motivation

Model reduction

- Developed by L. Engelhardt
- Degrees of freedom: $\sim 10^4$
- Calculation time: $\sim$ 30 seconds



Theoretical background: MCMC

$$\boldsymbol{y} = f(\boldsymbol{x}, \boldsymbol{e})$$

- **Statistical inverse problems**
  - All parameters are viewed as random variables:
    - $Y = f(X, E)$
  - Given:
    - $Y = \boldsymbol{y}_{\text{obs}}$
    - Prior probability density: $\pi_{\text{pr}}(\boldsymbol{x})$
    - Likelihood function: $\pi(\boldsymbol{y}|\boldsymbol{x})$
  - Searched:
    - Posterior probability density: $\pi_{\text{post}}(\boldsymbol{x})$

Theoretical background: MCMC

Theorem (Bayes' theorem of inverse problems)

Assume that the random variable $X \in \mathbb{R}^n$ has a known prior probability density $\pi(\boldsymbol{x})$ and the data consist of the observed value $\boldsymbol{y}_{obs}$ of an observable random variable $Y \in \mathbb{R}^k$ such that $\pi(\boldsymbol{y}_{obs}) > 0$. Then the posterior probability distribution of X, given the data $\boldsymbol{y}_{obs}$ is

$$\pi_{post}(\boldsymbol{x}) = \pi(\boldsymbol{x}|\boldsymbol{y}_{obs}) = \frac{\pi_{pr}(\boldsymbol{x})\pi(\boldsymbol{y}_{obs}|\boldsymbol{x})}{\pi(\boldsymbol{y}_{obs})}.$$

Theoretical background: MCMC

Looking at the Bayes' formula, we can say that solving an inverse problem may be broken into three subtasks:

- Based on all the prior information of the unknown X, find a prior probability density π_{pr} that reflects judiciously this prior information.
- Find the likelihood function $\pi(\boldsymbol{y}|\boldsymbol{x})$ that describes the interrelation between the observation and the unknown.
- Develop methods to explore the posterior probability density.

Metropolis-Hastings algorithm

1) Pick an initial value $\boldsymbol{x}_1 \in \mathbb{R}^n$ from the prior distribution and set $k = 1$.

2) Draw $\boldsymbol{y} \in \mathbb{R}^n$ from the proposal distribution $q(\cdot, \boldsymbol{x}_k)$ and calculate the acceptance ratio $\alpha(\boldsymbol{y}, \boldsymbol{x}_k) = \min\left(1, \frac{\pi(\boldsymbol{y})q(\boldsymbol{y},\boldsymbol{x})}{\pi(\boldsymbol{x})q(\boldsymbol{x},\boldsymbol{y})}\right)$.

3) Draw $t \in [0, 1]$ according the uniform probability distribution.

4) If $\alpha(\boldsymbol{y}, \boldsymbol{x}_k) \geq t$ set $\boldsymbol{x}_{k+1} = \boldsymbol{y}$ else $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k$. If the desired sample size is reached, stop the algorithm. Else increase $k \to k + 1$ and go to step 2.

Theoretical background: MCMC

Assumptions

- $Y = f(X, E) = \tilde{f}(X) + E$
- The random variable E has expectation value zero.
- The probability density π_{noise} of the noise is known.

$$\Rightarrow \pi(\boldsymbol{y}|\boldsymbol{x}) = \pi_{noise}(\tilde{f}(\boldsymbol{x}) - \boldsymbol{y})$$

Bayes' theorem:

$$\pi_{\text{post}}(\boldsymbol{x}) = \pi(\boldsymbol{x}|\boldsymbol{y}) = \frac{\pi_{\text{prior}}(\boldsymbol{x})\pi_{\text{noise}}(\tilde{f}(\boldsymbol{x}) - \boldsymbol{y})}{\pi(\boldsymbol{y})}$$

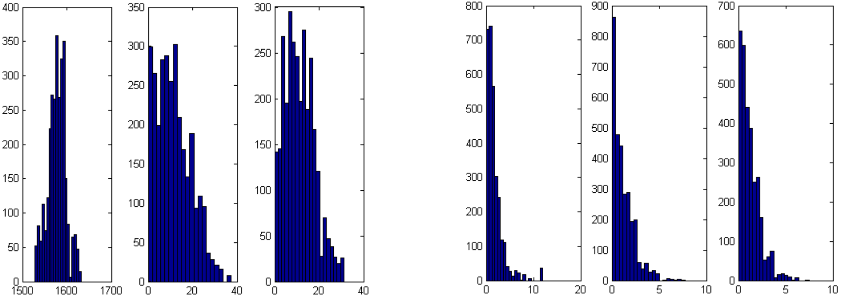
Theoretical background: MCMC

Priori distribution: Uniform distribution on the domain.

Proposal distribution: Normal distribution, where the mean should be the current state of the Markov chain.

Distribution of the error: Normally distributed, with expectation value zero.

First results

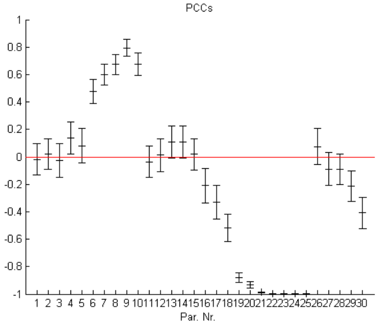


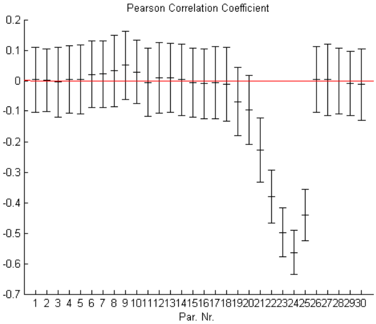
Sensitivity analysis

Sample based sensitivity analysis:

- Latin Hypercube sampling
- Correlation control
- Correlation coefficients:
  - **CC:** Pearson correlation coefficient
  - **PCC:** Partial correlation coefficient
  - **SRC:** Standardized regression coefficient
  - **RCC:** Spearman rank correlation coefficient
  - **PRCC:** Partial rank correlation coefficient
  - **SRRC:** Standardized rang regression coefficient
- Bootstrap confidence interval

Sensitivity analysis





Sensitivity analysis

Sensitive parameter:

- **Damping coefficients:**
  - Actuators 3 LE, 4 LE, 4 TE, 5 LE and 5 TE
- **Stiffness parameters:**
  - Actuators 2 TE, 3 LE, 3 TE, 4 LE, 4 TE, 5 LE and 5 TE

Copulas

A n-dimensional **copula** C is a distribution function on $[0,1]^n$ whose marginals are uniform distributions on $[0,1]$.

Theorem (Sklar)

Let F be an n-dimensional distribution function with margins $F_1, \ldots, F_n$. Then there exists an n-copula C such that for all $\boldsymbol{x} \in \mathbb{R}^n$

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)).$$

Copulas

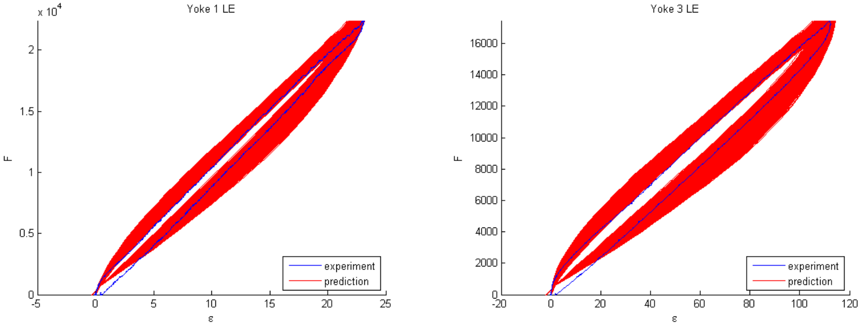
$$\boldsymbol{u} \sim C \Rightarrow \boldsymbol{x} = (F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)) \sim F$$

Definition (Gaussian copula)

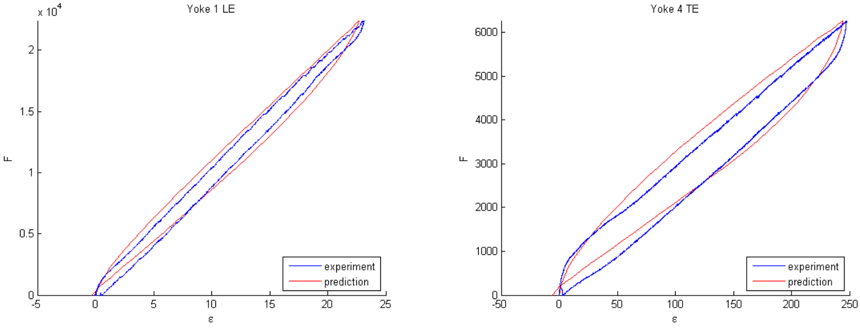
For a given correlation matrix $\boldsymbol{R} \in \mathbb{R}^{n \times n}$ the Gaussian copula with parameter matrix $\boldsymbol{R}$ is defined by:

$$C_R^{\text{Gauß}}(\boldsymbol{u}) = \Phi_{\boldsymbol{R}}(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)).$$

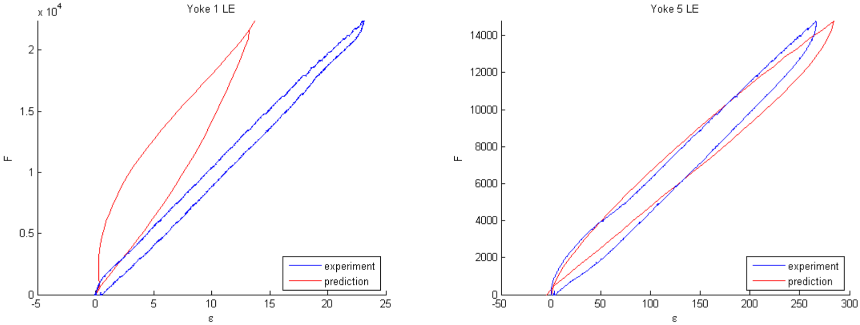
Stochastic error analysis



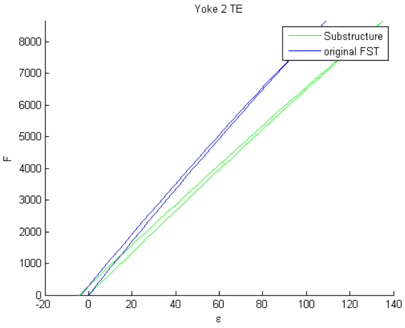
Stochastic error analysis

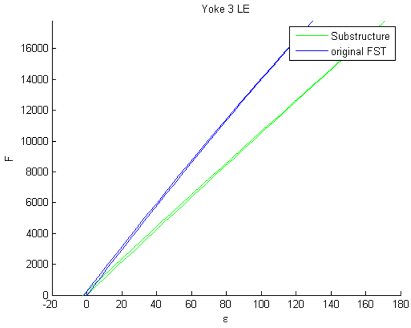


Application to the full scale test



Substructure



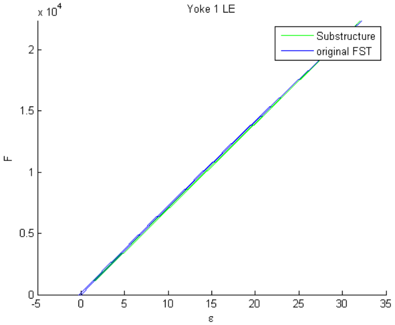


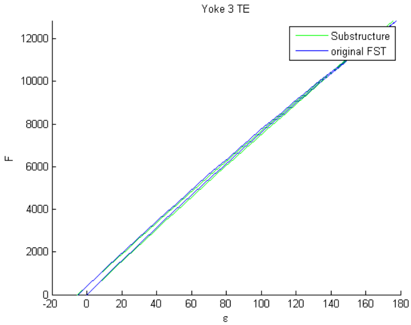
Substructure

Calibration of material parameters

*MATERIAL, NAME=MAT_MID_19
*ELASTIC, TYPE=LAMINA
** E_1, E_2, ν_{12}, G_{12}, G_{13}, G_{23}
62000., 62000., 0.05, 4200., 4200., 4200.
*DENSITY
1.6691e-06,
*EXPANSION, TYPE=ORTHO, ZERO=0.
2.3e-06, 2.3e-06

Substructure





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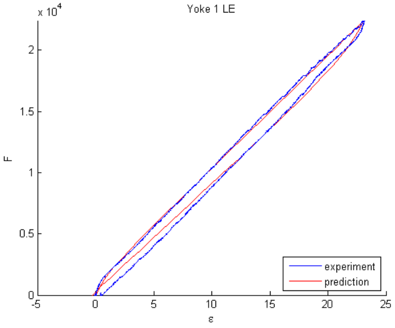

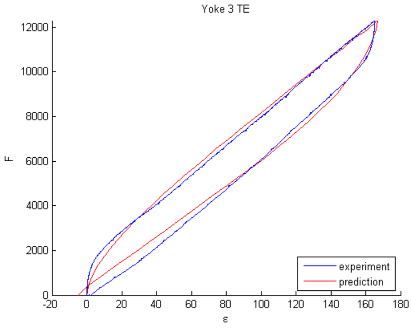

Application to the full scale test