Stochastic Fourier Integral Operators for Parameter Estimation and Damage detection.

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The (3×3)-system of linear, homogeneous, isotropic elasticity:

$$\partial_t^2 \boldsymbol{u} = \frac{\mu}{\rho} \Delta \boldsymbol{u} + \frac{\lambda + \mu}{\rho} \nabla \times \nabla \boldsymbol{u} + \boldsymbol{f}$$

for the displacement $\boldsymbol{u}(\boldsymbol{x}, t)$ with the density ρ and the Lamé constants λ, μ.

Wave Propagation in 3D Linear Elasticity

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Focus on full space problem with acting body force $\boldsymbol{f}(\boldsymbol{x}, t)$. As initial state we assume $\boldsymbol{u}(\boldsymbol{x}, 0) = \boldsymbol{0}$ and $\boldsymbol{u}_t(\boldsymbol{x}, 0) = \boldsymbol{0}$.

Lamé Representation (1)

Equation of motion

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There exist unique potentials Φ, $\boldsymbol{\Psi}$ such that

$$\boldsymbol{u} = \nabla \Phi + \nabla \times \boldsymbol{\Psi}, \qquad \nabla \cdot \boldsymbol{\Psi} = 0$$

The potentials satisfy a system of decoupled wave equations

$$\partial_{tt} \Phi - c_l^2 \Delta \Phi = \varphi, \qquad \partial_{tt} \Psi_j - c_s^2 \Delta \Psi_j = \psi_j, \quad j = 1, 2, 3$$

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with the lateral and transversal wave speeds given by

$$c_l^2 = \frac{\lambda + 2\mu}{\rho}, \qquad c_s^2 = \frac{\mu}{\rho}$$

Representing the Potentials by FIOs

The scalar wave equation

$$\partial_t^2 u - c^2 \Delta u = f, \qquad u\Big|_{t=0} = \partial_t u\Big|_{t=0} = 0$$

is transformed to a (decoupled) first order system of pseudodifferential operators by setting $u_\pm = (\partial_t \pm ic\sqrt{-\Delta})u$, resulting in

$$\partial_t u_\pm \mp ic\sqrt{-\Delta} u_\pm = f$$

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The solution is given by the FIO representation

$$u_\pm(x,t) = \int_{\mathbb{R}^3}\int_0^t \mathrm{e}^{\mathrm{i}(x\xi \pm c|\xi|(t-s))}\widehat{f}(\xi,s)\,\mathrm{d}s\,\mathrm{đ}\xi$$

and finally

$$u(x,t) = \frac{1}{2}\int_0^t \big(u_+(x,s) + u_-(x,s)\big)\,\mathrm{d}s$$

Parameter estimation

Fourier integral operators for parameter estimation:

- Unknown Lamé parameters.

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- Sensors measure time dependent signal (i.e. amplitude)
- Fourier integral operator solution in signal location.
- Calibrate Lamé parameters s.t. L_2-difference of signals is minimized.

2D-EXAMPLE

Let

$$\begin{cases} \partial_{tt}\boldsymbol{u} = \dfrac{1}{\rho}\mu\Delta\boldsymbol{u} + (\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u}) + \boldsymbol{f} \\ \boldsymbol{u}\big|_{t=0} = 0 \\ \partial_t\boldsymbol{u}\big|_{t=0} = 0 \end{cases}$$

where $\boldsymbol{f} = [\ 0, \widetilde{\delta}(x)\widetilde{\delta}(y)\sin(2\pi t)\chi_{[0,2\pi]}(t), 0\ ]^{\mathrm{T}}$.

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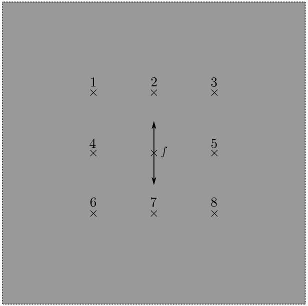
We assume sensores around the center give time dependent signal i.e. $h_i(t) := u(x_i, y_i, z_i, t), i = 1\ldots 8$

2D Example

Using FIOs for parameter estimation:

- Measurement is represented with FEM-signal $f_{FEM}(t)$ in sensor location with unknown (E_{FEM}, ν_{FEM}).
  - 256×256 elements (approx 15 min).
  - Abaqus, implicit time integration.



2D Example

Using FIOs for parameter estimation:

- Measurement is represented with FEM-signal $f_{FEM}(t)$ in sensor location with unknown (E_{FEM}, ν_{FEM}).
- FIO-solution $f_{FIO}(t)$ in sensor location with known (E_{FEM}, ν_{FEM}).
  - 256×256 grid.
  - 3 seconds to evaluate.

2D Example

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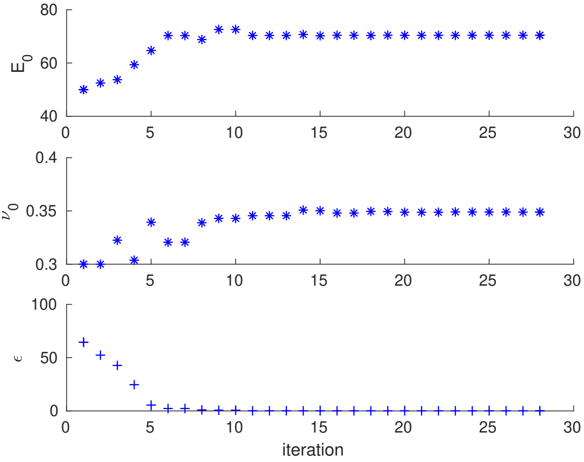
- FIO-solution $f_{FIO}(t)$ in sensor location with known (E_{FEM}, ν_{FEM}).
- Minimize

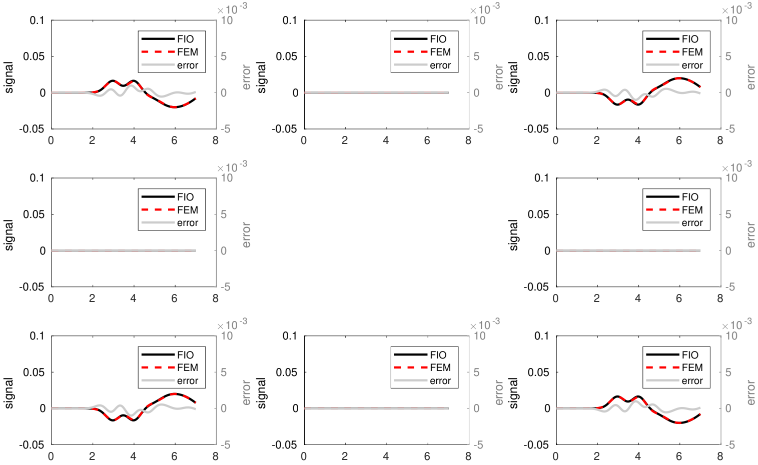
$$\|f_{FEM} - f_{FIO}(\cdot, E_{FIO}, \nu_{FIO})\|_{L_2}.$$

2D Example

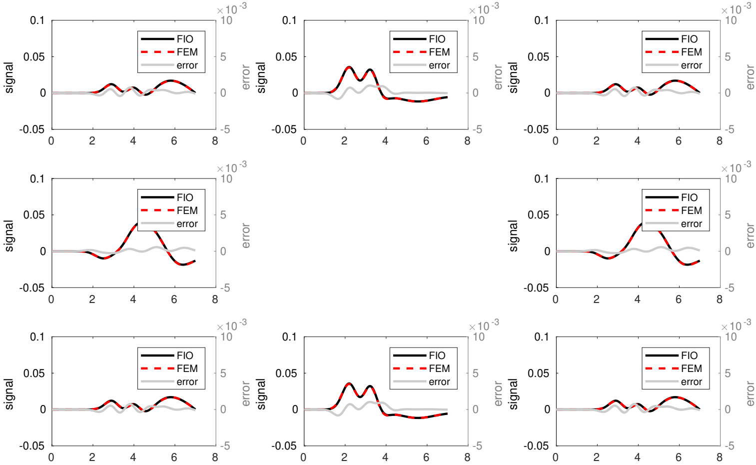


	Youngs Modulus	Poisson ratio
FEM	70.00 GPa	0.3500
optimized	70.41 GPa	0.3489

2D Example

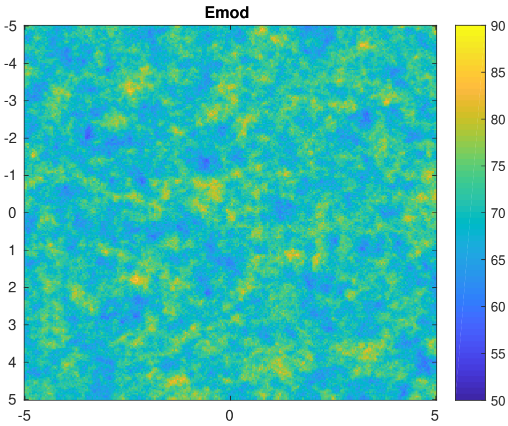


2D Example



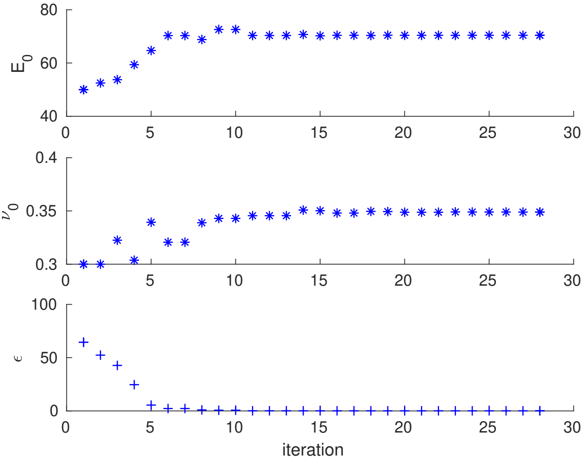
What if Lamé parameters are not "clean"?

2D Example



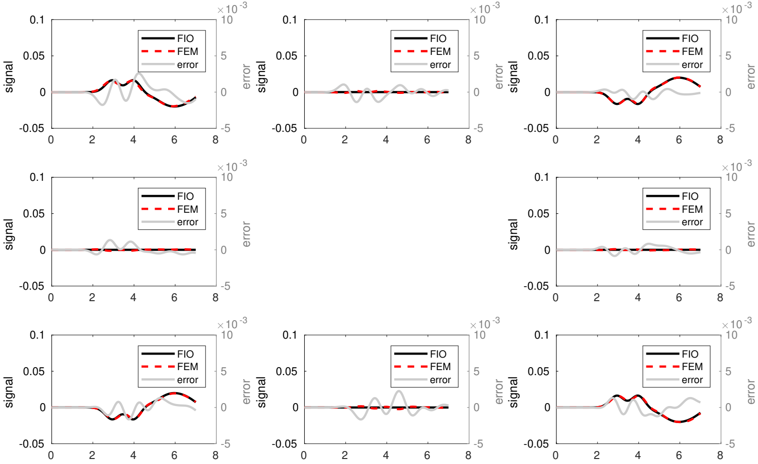
Perturdbed random field in FEM Model (5% deviation).

2D Example

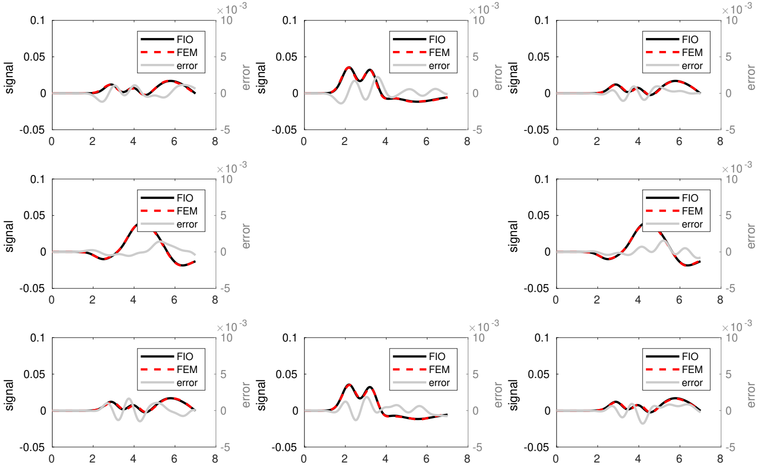


	Youngs Modulus	Poisson ratio
FEM nominal value	70.00 GPa	0.3500
optimized	70.97 GPa	0.3510

2D Example



2D Example



Also possible:

- Variance estimation of FEM random field:
  Compare variance of estimated parameters with variance of random field.

Parameter estimation

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  Compare variance of estimated parameters with variance of random field.

- Autocorrelation estimation of FEM random field:
  Compare correlation of estimated parameters with correlation length of random field.

*with some limitations

- The usual FIO is of form:

$$\int e^{ix\cdot\xi \pm ic\|\xi\| t}\, \widehat{u}(\xi)\, đ\xi$$

Damage detection

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- Add random perturbations:

$$\int e^{ix\cdot\xi \pm i(c+r(x))\|\xi\| t}\,(1 + r_2(x,\xi))\,\widehat{u}(\xi)\,đ\xi.$$

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- Generate Monte Carlo sample and test if measurement differs too much from sample.

2D example

- Same setup as before.

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- Random perturbation of FIO 5% of nominal value. Using random field.

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- Random perturbation of FIO 5% of nominal value. Using random field. That means

$$\int \mathrm{e}^{ix\cdot y \pm c\|\xi\| t}\widehat{u}(\xi)\mathrm{đ}\xi$$

becomes

$$\int \mathrm{e}^{ix\cdot y \pm c_{\omega}(x)\|\xi\| t}\widehat{u}(\xi)\mathrm{đ}\xi.$$

2D EXAMPLE

- Same setup as before.
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- Select features of signal:
    - Phase angle of dominant frequencies
    - Amplitude of dominant frequencies

2D example

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- Random perturbation of FIO 5% of nominal value. Using random field.
- Select features of signal:
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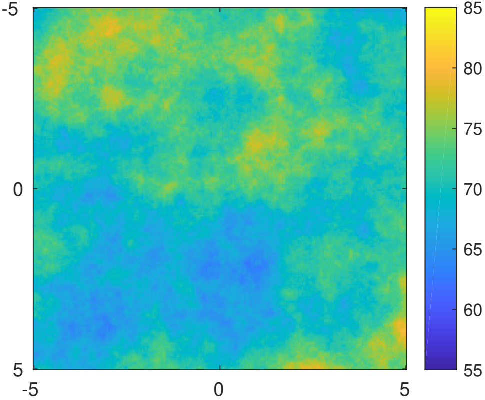
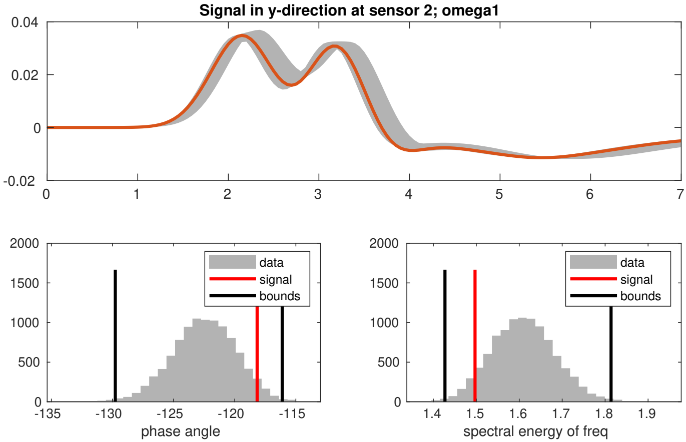
  - Amplitude of dominant frequencies
- 4 scenarios:
  1. Material ok
  2. Youngs modulus too small
  3. Youngs modulus too large
  4. Crack (set Young's modulus to zero)

2D EXAMPLE



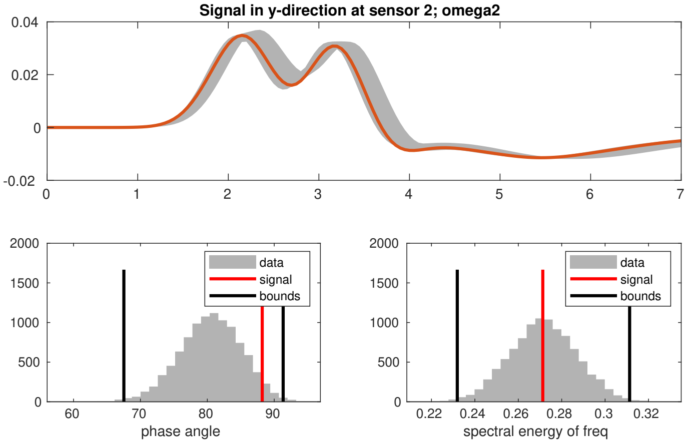
Material "ok"

2D EXAMPLE



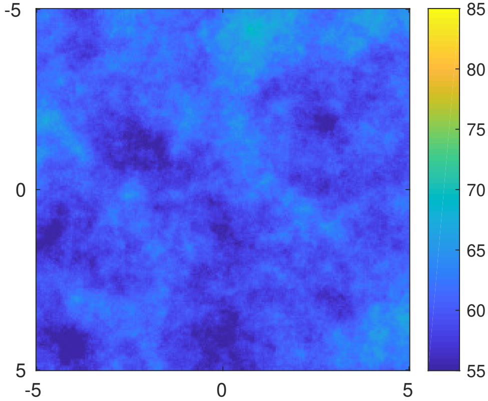
Signal in y-direction at sensor 2; omega1
0.04
0.02
0
-0.02
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
-135
-130
-125
-120
-115
phase angle
data
signal
bounds
2000
1500
1000
500
0
1.4
1.5
1.6
1.7
1.8
1.9
spectral energy of freq
data
signal
bounds

2D EXAMPLE



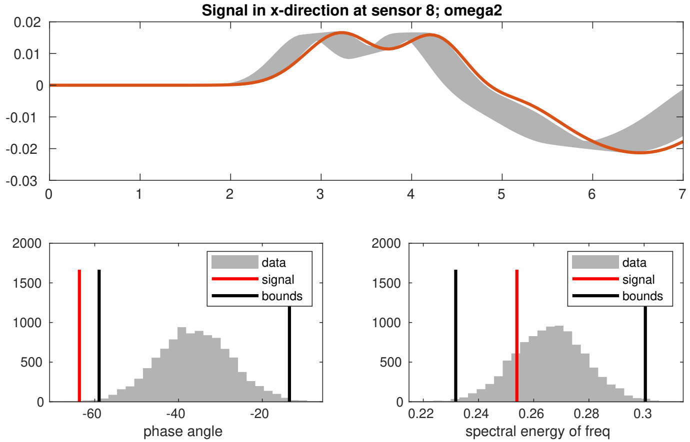
Signal in y-direction at sensor 2; omega2
0.04
0.02
0
-0.02
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
data
signal
bounds
60
70
80
90
phase angle
2000
1500
1000
500
0
data
signal
bounds
0.22
0.24
0.26
0.28
0.3
0.32
spectral energy of freq

2D example

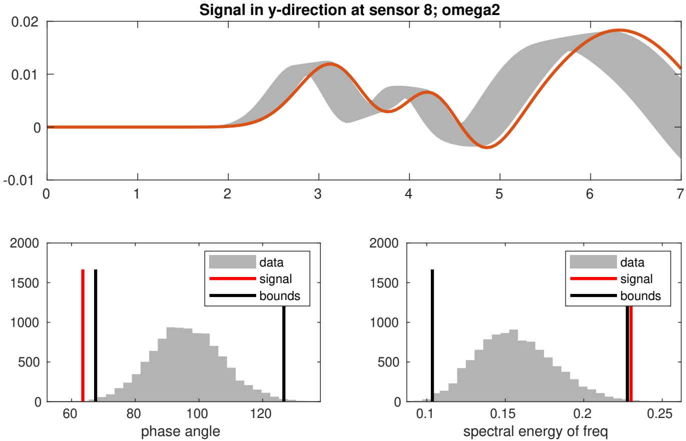


Young's modulus too small

2D EXAMPLE


Signal in x-direction at sensor 8; omega2
0.02
0.01
0
-0.01
-0.02
-0.03
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
-60
-40
-20
phase angle
data
signal
bounds
2000
1500
1000
500
0
0.22
0.24
0.26
0.28
0.3
spectral energy of freq
data
signal
bounds

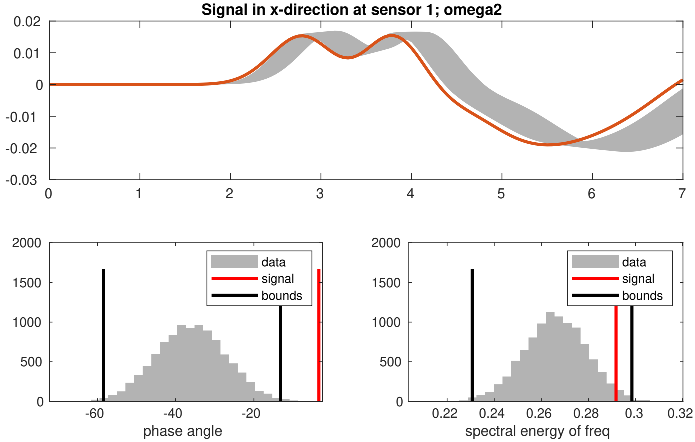
2D EXAMPLE


Signal in y-direction at sensor 8; omega2
0.02
0.01
0
-0.01
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
60
80
100
120
phase angle
data
signal
bounds
2000
1500
1000
500
0
0.1
0.15
0.2
0.25
spectral energy of freq
data
signal
bounds

2D EXAMPLE

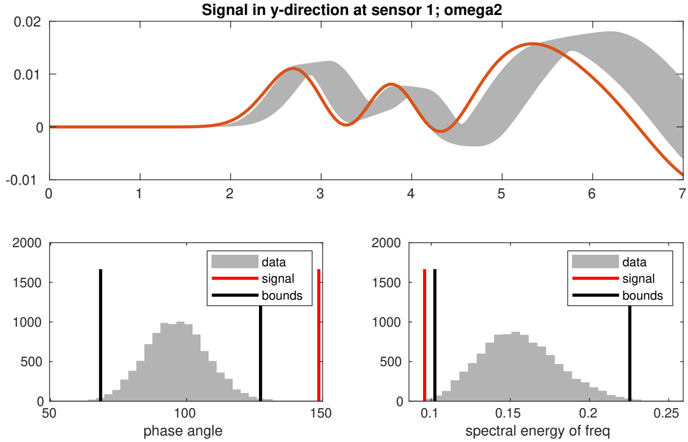
Young's modulus too large

2D EXAMPLE



Signal in x-direction at sensor 1; omega2
0.02
0.01
0
-0.01
-0.02
-0.03
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
-60
-40
-20
phase angle
data
signal
bounds
0.22
0.24
0.26
0.28
0.3
0.32
spectral energy of freq

2D example


Signal in y-direction at sensor 1; omega2
0.02
0.01
0
-0.01
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
50
100
150
phase angle
data
signal
bounds
2000
1500
1000
500
0
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0.15
0.2
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spectral energy of freq
data
signal
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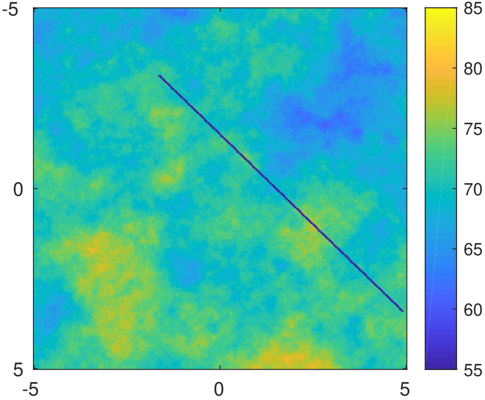
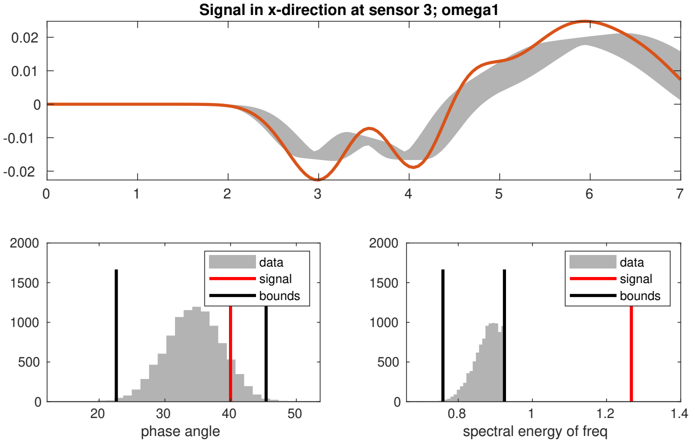
2D EXAMPLE



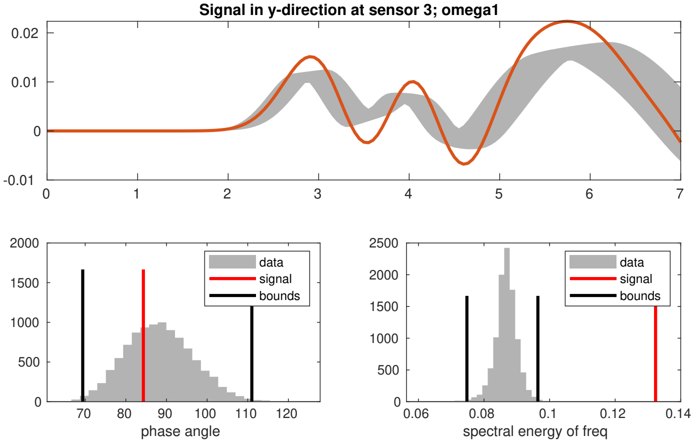
Crack

2D EXAMPLE



Signal in x-direction at sensor 3; omega1
0.02
0.01
0
-0.01
-0.02
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
20
30
40
50
phase angle
data
signal
bounds
2000
1500
1000
500
0
0.8
1
1.2
1.4
spectral energy of freq
data
signal
bounds

2D example



Signal in y-direction at sensor 3; omega1
0.02
0.01
0
-0.01
0
1
2
3
4
5
6
7
2000
1500
1000
500
0
70
80
90
100
110
120
phase angle
data
signal
bounds
2500
2000
1500
1000
500
0
0.06
0.08
0.1
0.12
0.14
spectral energy of freq
data
signal
bounds

Testing

We implement three tests. The null-hypothesises are

- phase angle of dominating frequencies are larger than critical angle (right sided test).

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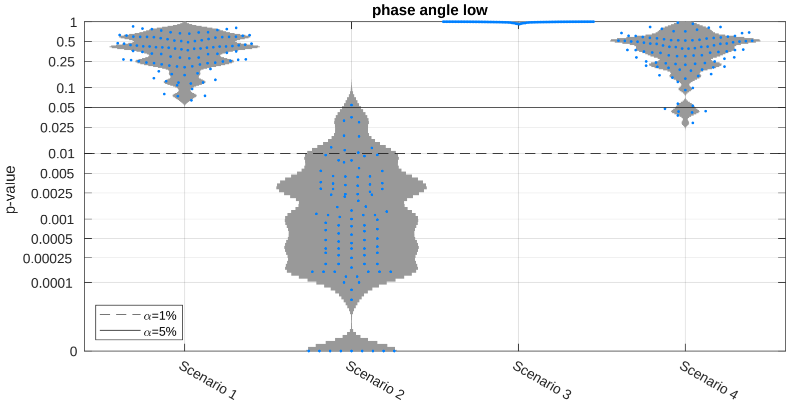
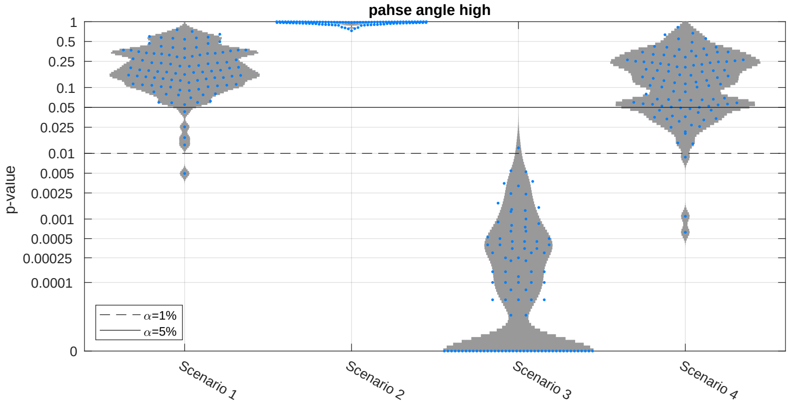
- spectral energy density is in symmetric interval around mean.

Monte Carlo simulation of all 4 scenarios with sample size $N = 100$. And apply all 3 tests.

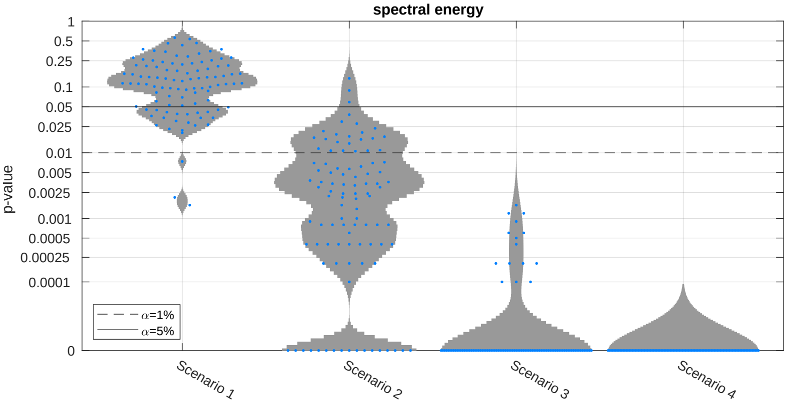
Testing results



Testing results



Testing results



Summary and Outlook

Summary:

- FIOs describing elastic waves in solids
- (Stochastic) parameter estimation.
- Stochastic FIOs for damage detection

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Outlook:

- Surface waves / surface induced waves
- Non-isotropic case

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Thank you!

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